The Constructive in Logic and Applications

A Conference in Honor of the 60th Birthday of Sergei Artemov

May 23 – May 25, 2012

The Graduate Center, City University of New York

Abstracts of Talks

Condensing Theories May 23rd
Gerald Sacks (Harvard & MIT) 9:15 - 10:00

Some results in infinitary model theory relating stability and Gödel condensation.

Solvers - applications of logic at the beginning of the 21st Century May 23rd
Victor W. Marek (University of Kentucky) 10:00 - 10:45

We discuss solvers, a class of computer programs based on various logic formalisms and allowing for describing and solving a number of constraint satisfaction problems.

The specific classes of solvers we focus on are: SAT solvers (based on classical propositional logic) and ASP solvers (based on stable semantics of logic programs and related to Gödel - Smetanich logic).

Which Quantifiers are logical? A combined semantical and inferential criterion May 23rd 11:00 - 11:45
Solomon Feferman (Stanford)

The aim of logic is to characterize the forms of reasoning that lead invariably from true sentences to true sentences, independently of the subject matter; thus its concerns combine semantical and inferential notions in an essential way. Up to now most proposed characterizations of logicality of sentence generating operations have been given either in semantical or inferential terms. This paper offers a combined semantical and inferential criterion for logicality (improving one originally proposed by Jeffery Zucker) and shows that any quantifier that is to be counted as logical according to that criterion is definable in first order logic.

Unknowable Mathematical Truths, Natural Questions, and the Π_1 lattice May 23rd 11:45 - 12:30
Haim Gaifman (Columbia)

A classical view of constructive semantics May 23rd
Sergei Artemov (CUNY) 2:00 - 2:45

In constructive logic, a sentence is true if it has a proof. This paradigm led to the well-known Brouwer–Heyting–Kolmogorov (BHK) semantics of proofs, which can be viewed as a means of defining constructive semantics within the framework of classical mathematics. This approach was explicit in Kolmogorov's works and similar ideas were expressed by Gödel, who in the 1930s initiated the provability logic project in order to find a classical specification of constructive reasoning. The original version of Gödel's project considered the propositional case and was completed with the Logic of Proofs (1995). In this talk, we extend Gödel's aforementioned project to the first-order case and offer a formalization of the first-order BHK within the classical first-order logic of proofs FOLP, as found in a recent joint paper with Tatiana Yavorskaya.

New May 23rd
Dexter Kozen (Cornell) 2:45 - 3:30

There are many situations in computing in which we want to create something new. We often do not care exactly what is created, as long as it has the right properties. For example, when allocating a new heap cell, we do not care exactly what its address in memory is, but only that we can store and retrieve data there; for that purpose, any heap cell is as good as any other. In object-oriented programming, when we create a new object, we only care that it has the right fields and methods and is different from every other object previously created. In the lambda-calculus, when we rename a bound variable, we do not care what the new variable is as long as it is fresh.

As common as it is, the intuitive act of creating a new object out of nothing does not fit well with set-theoretic foundations. Such situations are commonly modeled as an allocation of one of a previously existing collection of equivalent candidates.

There are two difficulties with this view: the candidates for allocation should be indiscernible, but they cannot be if one must be chosen deterministically; and cardinality restrictions on the set of candidates
often interfere with closure conditions on the language.

In this talk I will describe a theoretical device for modeling the creation of new indiscernible semantic objects during program execution. Simply put, a semantic object is created by allocating a name for it. The object itself is defined to be the congruence class of all its names. I will describe an operational semantics for a higher-order functional language with imperative and object-oriented features implementing this idea.

The Dynamic Logic of Justified Knowledge and Belief May 23rd
Bryan Renne (Universities of Amsterdam & of British Columbia) 3:45 - 4:15

I will present recent work joint with Alexandru Baltag and Sonja Smets on a logic for reasoning about justified belief change, soft evidence, and defeasible knowledge. This logic combines evidential machinery from Justification Logic with Dynamic Epistemic Logic-based belief dynamics to characterize certain kinds of evidence-based reasoning on models from Belief Revision theory. I will explain how this framework provides us with a dynamic take on one of Lehrer's Nogot-Havit Gettier scenarios.

Invariant Maximality May 23rd
Harvey Friedman (Ohio State) 4:15 - 5:00

Let $h_1,\ldots,h_n$ be partial functions from $Q_{\geq 0}$ into $Q_{\geq 0}$. We investigate "every order invariant graph on $Q^k_{\geq 0}$ has a diagonally $h_1,\ldots,h_n$ invariant maximal clique." Even for particularly simple easily presented order theoretic h's, this results in statements provably equivalent to the consistency of certain large cardinal hypotheses.

Intersection Types in Constructive Type Theory May 24th
Robert Constable (Cornell) 9:00 - 9:45

The intended semantics for intuitionistic first-order logic (iFOL) is often called the Brouwer-Heyting-Kolmogorov (BHK) interpretation. Most of the positive semantic results about this logic, both classical and intuitionistic, are given in terms of Beth and Kripke models. The negative results depend on a notion of intuitionistic validity in which the definition of a model is not canonical; however, the BHK interpretation defines the logical operators. These negative results suggest that completeness is not achievable with this semantics since given Church's Thesis (CT), the valid formulas of iFOL are not r.e.

Using extensional constructive type theory (CTT) as the semantic model allows a very precise and canonical semantics for which positive results are possible. I will lay the ground work to state precisely (but not prove in this lecture) one such completeness result. It depends on using the rich type system of CTT to capture more than one version of the BHK semantics, and it benefits from not using CT. These ideas also illustrate that the BHK semantics can be made precise and clear in a language that is natural to both constructive and classical mathematicians. It is the basis for a classical evidence semantics for FOL with respect to which it is complete.

Collaborative Systems May 24th
Andre Scedrov (U Penn) 9:45 -10:30

Our earlier work with Kanovich and Rowe introduced a model of collaboration, in which the participants are unwilling to share all their information with each other, but some information sharing is unavoidable when achieving a common goal. The need to share information and the desire to keep it confidential are two competing notions which affect the outcome of a collaboration. Our model is based on the notion of a plan which originates in the AI literature. Here we consider two extensions of the model. The first extension allows for updates of values with fresh ones, such as updating a password. The second extension allows for the specification of policies and systems with explicit time. Time is discrete and is specified by timestamps attached to facts. Actions, goal and critical states may be constrained by means of relative time constraints. All the players inside our system, including potential adversaries, have similar capabilities. They have bounded storage capacity, that is, they can only remember a bounded number of facts. This is technically imposed by allowing only the so-called balanced actions, that is, actions that have the same number of facts in their pre and post conditions. We investigate the complexity of the planning problem, whether the players can reach a goal while avoiding certain critical configurations along the way. We show that this problem is PSPACE-complete. This is joint work with T. Ban Kirigin, M. Kanovich, V. Nigam, R. Perovic, and C. Talcott.

'At Most One' constructively May 24th
Joan Moschovakis (Occidental College & MPLA in Athens) 10:45 - 11:30

This "housekeeping" talk clarifies a few points in the axiomatization and formalization of constructive and intuitionistic analysis, especially: What does “there is at most one x in X satisfying P(x)” mean constructively if X is the set of all natural numbers, or the set of all binary sequences, or Baire space, or a canonical set of real-number generators, or any constructive measure space?

Two constructively different (but classically equivalent) answers lead to two versions WKL!, WKL!! of weak König's Lemma with a uniqueness hypothesis. While WKL! is constructively equivalent to the decidable fan theorem (Ishihara, Berger), WKL!! is (recursively realizable but) independent of intuitionistic analysis and weaker than WKL, which follows constructively from KL(Sigma-0-1)!!.

Intrinsic complexity in arithmetic and algebra May 24th
Yiannis Moschovakis (UCLA & University of Athens) 11:30 - 12:15

Sample result: With each structure

$(A,\Phi) = (A,f_1,...,f_k,R_1,...,R_m)$,

each n-ary relation P on the universe A and each tuple x in A^n, we will associate a number

$m = \text{calls}(A,\Phi,P,x) =$ the intrinsic calls-complexity of P in (A,Φ) at x

such that if any algorithm alpha from the primitives Phi decides P, then alpha must execute at least m calls to the primitives on the input x. This is a theorem for concrete algorithms specified by the familiar models of computation, and it will be made plausible for all "algorithms from specified primitives" using ideas from abstract model theory.

For coprimeness in arithmetic (from various primitives) and polynomial testing for 0 from the field operations, the intrinsic calls-complexity gives the optimal or best known lower bound results.

Algorithmic Randomness via Probabilistic Algorithms May 24th
Sam Buss (UC San Diego) 2:00 - 2:45

This talk discusses probabilistic martingales, in which randomness is used to decide whether and how much to bet. We show that different versions of computable probabilistic martingales can be used to characterize ML-randomness, computable randomness, and partial computable randomness. Our characterization of ML-randomness partially addresses a critique of Schnorr's by formulating ML randomness in terms of a computable process rather than a computably enumerable function. This is joint work with Mia Minnes.

Feedback and partial traces in proof networks May 24th
Philip Scott (University of Ottawa) 2:45 - 3:30

The algebra of networks with feedback has recently seen a resurgence of interest among mathematicians, category theorists and theoretical computer scientists. In the 1990's, categories capturing general feedback and trace in wide areas of mathematics were introduced by Joyal, Street, and Verity (and independently in network algebra, by G. Stefanescu). Such categories (traced monoidal categories) also turned out to model information flow in proof theory (e.g. in Girard's Geometry of Interaction program). More recently, these structures have arisen in several areas: quantum programming languages, computational biology, relational data flow, Hoare logics for linear systems, etc. Recently, a new extension has arisen (partially traced categories) with intrinsic connections to several of the above areas (e.g. proof theory and quantum computing), and seemingly of general mathematical interest. We shall survey some of these developments.

The Logic of Uncertain Justifications May 24th
Robert Milnikel (Kenyon) 3:45 - 4:15

There is a long tradition in epistemic logic of formalizing the notion of "degree of belief," but seldom were the reasons behind those beliefs incorporated in the formal system. Artemov's Justification Logic provides a framework for reasoning about the reasons behind beliefs. In this short talk, we present an adaptation of Justification Logic which allows for justifications of varying reliability.

Sergei Artemov from 5 to 60 May 24th
Anil Nerode (Cornell) 4:15 - 5:00

A brief biographical sketch with pictures.

From Positive to Zero to Negative Probability May 25th
Adam Brandenburger (NYU) 9:00 - 9:45

Hybrid Answer Set Programming May 25th
Jeff Remmel (UC San Diego) 9:45 -10:30

We will describe an extension of the Answer Set Programming paradigm called Hybrid Answer Set Programming (H-ASP). H-ASP allows the user to combine numerical and probabilistic algorithms which are needed to realistically simulate physical processes and the expressive power of Answer Set Programming which provides the ability to elegantly model laws of the system.

This is joint work with Alex Brik.

Geodesic spaces: Euclid's five postulates as an equational theory, starting with the second May 25th
10:45 - 11:30
Vaughan Pratt (Stanford)

Whereas Descartes' analytic geometry is nowadays formalized algebraically in terms of the equational theory of vector spaces, expandable with an inner product, it has long been assumed that Euclid's synthetic geometry must be formalized logically in the manner of Pasch, Hilbert, and Tarski, whose Frege-style axioms however lack the functional-programming constructivity of equational logic. We challenge this assumption with a purely equational axiomatization that reorders Euclid's postulates as 2,5,1,4,3, the first three of which cleanly isolate the affine part.

The second postulate, that every segment extends, is expressed with a binary extension operation satisfying $xx = (xy)y = x$. The fifth postulate, that inclined lines meet, is expressed via Varignon's theorem as the familiar interchange law $(wx)(yz) = (wy)(xz)$. The first postulate, generalized to the convex hull of n rather than just 2 points, is expressed with an n-ary centroid operation satisfying a conjugate pair of equations for each $n \geq 2$. We show that the resulting variety including its homomorphisms is equivalent to the category of affine spaces over the rationals, in apparent contradiction to the upward Loewenheim-Skolem theorem. The category of vector spaces over the rationals is obtained simply by expanding the language with a constant symbol O for the origin, without further equations.

To capture inner product, Euclid's fourth postulate concerning right angles is expressed with a ternary operation on triangles that rectifies them, while the third concerning circles uses two *partial* ternary operations giving respectively the nearest and tangent points on a circle. We show that every model of countable dimension at least two is orthonormalizable via the Gram-Schmidt process, and that the coordinates thereby associated to the terms of the language on their respective domains are precisely the constructible numbers.

Non-Euclidean geometry is treated analogously to dropping the abelian requirement on groups, namely by putting the fifth postulate in an equivalent form admitting a one-parameter family of natural variants, three of which yield models with positive Gaussian curvature and the rest negative.

Knowledge and Rationality May 25th

Round Table chaired by Rohit Parikh (CUNY) 11:30 - 12:30

Participants:
Rohit Parikh (BC and GC), Pierpaolo Battigalli (Bocconi University), Adam Brandenburger (NYU), Florian Lengyel (CCNY and GC), Yang Liu (Columbia), Tudor Protopopescu (GC)

The following three aspects of human activity are roughly related. They are reasoning, logic and rationality. And how are they related?

Reasoning is the process of taking information or facts which one has and arriving at conclusions which follow, where "follow" is left informal. *Logic* can be described as reasoning wearing a necktie. The notion of "what follows" is made precise by specifying some formal language in which the "information" is expressed and the "follows" is made precise by means of some syntactic rules.

Rationality is more related to practical goals and not just to reasoning. And yet it is typically the context in which reasoning takes place. An agent has certain goals and certain information. The agent also has choices of actions. Which action is best? This decision is made in terms of the information which the agent has, and a process of reasoning - formal or informal - is used to derive further information. The notion of practical reasoning, closely related to rationality, already goes back to Aristotle.

A formal version of rationality is *utility maximization*. This version assumes that an agent has a subjective probability over various outcomes associated with actions and also a cardinal utility assigned to the outcomes. This allows an expected utility to be assigned to varius actions and then an action which maximizes expected utility is chosen by a rational agent.

In the absence of cardinal utility, an agent still may have *ordinal utility* where the agent has preferences among outcomes but not a clear assisgnment of cardinal values to the outcomes nor perhaps a subjective probability. The agent may then "play safe" and choose an action whose outcome is guaranteed to have a certain minimum ordinal value. Or the agent may be ambitious and choose an action whose possible outcome is as good as possible. A young man too shy to ask for a date follows the first, maxmin strategy. A more forward young man may ask a strange young woman for a date without worrying about rejection.

Again, both reasoning and information adding up to knowledge or justified belief will enter at this stage. We will call an agent rational if he/she does the best - however defined - given the agent's epistemic situation.

Various names are associated with the early developments in this area. von Neumann and Morgenstern, Savage, Hintikka, Lewis, Schelling, Aumann, and many others. After roughly forty years of development, we now have something resembling a marriage between economics and logic, or if not a marriage, at least a friendship.

But the ultimate goal of this enterprise is intelligent decision making at both
the individual and social levels.

Domain Independence, Predicativity, and Constructivity May 25th

Arnon Avron (Tel-Aviv University) 2:00 - 2:45

We develop formal systems for predicative set theory, which are sufficient for applicative mathematics, and closely resemble real mathematical practice. Our framework is based on a general theory of domain independence and absoluteness in mathematics.

Realization Made Simpler May 25th
Melvin Fitting (CUNY) 2:45 - 3:30

Each justification logic is connected with a corresponding modal logic via a Realization Theorem. This says that, given a modal theorem, each modal operator can be replaced with an explicit justification term to produce a theorem of the related justification logic. In a sense, modal operators model implicit knowledge, while justification terms make the reasoning involved explicit. The Realization Theorem connecting S4 and LP supplies a key step in carrying out the BHK project. There are several proofs of the Realization Theorem, most of them constructive. The construction involves an induction on cut-free proof complexity, not on formula complexity. In this talk, we show how all this can be considerably simplified. We do so by introducing an intermediate step, a Quasi-Realization Theorem. Quasi-Realizations can be produced constructively, but much more simply than Realizations, from cut-free modal proofs. Their non-constructive existence can also be shown rather easily using the Model Existence Theorem, as we do in the talk. A Quasi-Realization is still sufficient for the BHK project. Quasi-Realizations can be converted into Realizations in a way that is uniform across several justification logics. It should be noted that all this extends naturally to the recently discovered first-order version of justification logics, though we do not discuss this in the talk.

On interchangeability of Nash equilibria in multi-player zero-sum games May 25th
3:30 - 4:00
Pavel Naumov (McDaniel)